B. Sc. Part I 2nd Paper

Diff. Calculus

Indeterminate Forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \times \infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0

L' Hospital's rule

$$\lim_{x \to a} \frac{f(x)}{\phi(x)} = \lim_{x \to a} \frac{f'(x)}{\phi'(x)}$$

provided the latter limit exists

1. Evaluate $\lim_{x \to 0} \frac{\sin x}{x}$.

Soln The given term $= \lim_{x \to 0} \frac{\sin x}{x}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)}$$

$$= \lim_{x \to 0} \cos x = 1$$

2. Evaluate $\lim_{x \to 1} \frac{\log x}{x - 1}$.

Soln $\lim_{x \to 1} \frac{\log x}{x - 1}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 1} \frac{\frac{d}{dx}(\log x)}{\frac{d}{dx}(x - 1)} = \lim_{x \to 1} \frac{\frac{1}{x}}{1} = \lim_{x \to 1} \frac{1}{x} = 1.$$

3. $\lim_{x\to 0} \frac{\sin 5x}{\sin 2x}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x\to 0} \frac{\frac{d}{dx}(\sin 5x)}{\frac{d}{dx}(\sin 2x)}$

$= \lim_{x\to 0} \frac{5\cos 5x}{2\cos 2x} = \frac{5}{2} \cdot \frac{\lim_{x\to 0} \cos 5x}{\lim_{x\to 0} \cos 2x}$

$= \frac{5}{2} \times \frac{1}{1} = \frac{5}{2}$.

4. $\lim_{x\to 0} \frac{a^x - 1}{x}$ $\left[\frac{0}{0}\right]$

$= \lim_{x\to 0} \frac{\frac{d}{dx}(a^x - 1)}{\frac{d}{dx}(x)}$

$= \lim_{x\to 0} \frac{a^x \log a}{1} = a^0 \log a = \log_e a$

5. $\lim_{x\to 0} \frac{x - \tan x}{x^3}$ $\left[\frac{0}{0}\right]$

$= \lim_{x\to 0} \frac{\frac{d}{dx}(x - \tan x)}{\frac{d}{dx}(x^3)} = \lim_{x\to 0} \frac{1 - \sec^2 x}{3x^2}$ $\left[\frac{0}{0}\right]$

$= \lim_{x\to 0} \frac{\frac{d}{dx}(1 - \sec^2 x)}{\frac{d}{dx}(3x^2)} = \lim_{x\to 0} \frac{-2\sec^2 x \tan x}{6x}$ $\left[\frac{0}{0}\right]$

$= \lim_{x\to 0} \frac{-2(2\sec^2 x \tan x + \sec x)}{6} = \frac{-2}{6}(2\times 0 + 1) = -\frac{1}{3}$